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JHEP05(2009)009

(http://iopscience.iop.org/1126-6708/2009/05/009)

Published by IOP Publishing for SISSA

Received: *March 27, 2009*
Accepted: *April 16, 2009*
Published: *May 5, 2009*

JHEP05(2009)009

Janus field theories from non-linear BF theories for multiple M2-branes

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Abstract: We integrate the nonpropagating B_μ gauge field for the non-linear BF Lagrangian describing N M2-branes which includes terms with even number of the totally antisymmetric tensor M^{IJK} in arXiv:0808.2473 and for the two-types of non-linear BF Lagrangians which include terms with odd number of M^{IJK} as well in arXiv:0809:0985. For the former Lagrangian we derive directly the DBI-type Lagrangian expressed by the SU(N) dynamical A_μ gauge field with a spacetime dependent coupling constant, while for the low-energy expansions of the latter Lagrangians the B_μ integration is iteratively performed. The derived Janus field theory Lagrangians are compared.

Keywords: Chern-Simons Theories, M-Theory

ArXiv ePrint: 0903.4500

 doi:10.1088/1126-6708/2009/05/009

Contents

1 Introduction

Inspired by Bagger and Lambert [1] and Gustavsson [2] (BLG) who constructed the worldvolume theory of multiple coincident M2-branes following earlier works [3, 4], the multiple M2-branes have been extensively studied. The BLG theory is described by a three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons gauge theory with manifest SO(8) R-symmetry based on 3-algebra with a positive definite metric, that is, the unique nontrivial $\mathcal{A}_4$ algebra [5]. However, this Chern-Simons gauge theory expresses two M2-branes on a R^8/Z_2 orbifold [6].

A class of models based on 3-algebra with a Lorentzian metric have been constructed by three groups [7–9] where the low-energy worldvolume Lagrangian of N M2-branes in flat spacetime is described by a three-dimensional superconformal BF theory for the $su(N)$ Lie algebra. Using a novel Higgs mechanism of ref. [10] the BF membrane theory has been shown to reduce to the three-dimensional maximally supersymmetric Yang-Mills theory whose gauge coupling is the vev of one of the scalar fields [7, 9, 11]. For the prescription of the ghost-like scalar fields a ghost-free formulation has been proposed by introducing a new gauge field for gauging a shift symmetry and then making the gauge choice for decoupling the ghost state [12–14]. In ref. [15] starting from the maximally supersymmetric three-dimensional Yang-Mills theory and using a non-Abelian duality transformation due to de Wit, Nicolai and Samtleben (dNS) [16], the Lorentzian BLG theory has been reproduced.

The relation between the $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory [17] and the $\mathcal{N} = 8$ Lorentzian BLG theory has been studied [18–21]. The various investigations related with the BLG theory have been performed [22–25]

There has been a construction of a manifestly SO(8) invariant non-linear BF Lagrangian for describing the non-Abelian dynamics of the bosonic degrees of freedom of N coincident M2-branes in flat spacetime, which reduces to the bosonic part of the BF membrane theory for SU(N) group at low energies [26]. This non-linear Lagrangian is an extension of the non-Abelian DBI Lagrangian [27, 28] of N coincident D2-branes and includes only terms with even number of the totally antisymmetric tensor M^{IJK}. Further, two types of non-linear BF Lagrangians have been presented such that they include terms with even and odd number of M^{IJK} [29]. A different kind of non-linear gauged M2-brane Lagrangian has been proposed for the Abelian case [30].

As a related work, it has been shown that starting with the $\mathcal{N} = 8$ supersymmetric Yang-Mills theory on D2-branes and incorporating higher-derivative corrections to lowest nontrivial order, the Lorentzian BF membrane theory including a set of derivative corrections is constructed through a dNS duality [31] (see [32]). The higher-derivative corrections to the Euclidean $\mathcal{A}_4$ BLG theory have been determined [33] by means of the novel Higgs mechanism and also shown to match the result of [31]. The couplings of the worldvolume of multiple M2-branes to the antisymmetric background fluxes have been investigated by using the low-energy Lagrangian for multiple M2-branes [34, 35] as well as the non-linear BF Lagrangian [36]. There have been proposals for the non-linear Lagrangians for describing the M2-brane-anti-M2-brane system [37] and the unstable M3-brane [38].

We will perform the integration over the redundant B_μ gauge field for the non-linear BF Lagrangians of ref. [26] and ref. [29], to see how the Lagrangians are described by the dynamical A_μ gauge field. We will carry out the B_μ integration directly for the non-linear Lagrangian of ref. [26], while the B_μ integration will be iteratively performed for the two types of non-linear BF Lagrangians of ref. [29]. These three B_μ integrated Lagrangians will be compared.

2 Non-linear BF Lagrangian with even number of M^{IJK}

We consider the non-linear BF Lagrangian for SU(N) group which describes the non-Abelian dynamics of the bosonic degrees of freedom of N M2-branes in flat spacetime [26]

$$\begin{aligned} L = & -T_2 \mathrm{STr}\left(\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ} \tilde{D}_\nu X^J\right)} (\det \tilde{Q})^{1/4}\right) \\ & + \mathrm{Tr}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda}\right) + \left(\partial_\mu X^I_- - \mathrm{Tr}\left(X^I B_\mu\right)\right)\partial^\mu X^I_+ \\ & - \mathrm{Tr}\left(\frac{X_+\cdot X}{X_+^2}\hat{D}_\mu X^I \partial^\mu X^I_+ - \frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2 \partial_\mu X^I_+ \partial^\mu X^I_+\right), \end{aligned} \tag{2.1}$$

where $X_+^2 = X^I_+ X^I_+$ and the M2-brane tension T_2 is related to the eleven-dimensional Planck length scale l_p as $T_2 = 1/(2\pi)^2 l_p^3$. The two non-dynamical gauge fields A_μ, B_μ and the scalar fields X^I ($I = 1, \ldots, 8$) are in the adjoint representation of SU(N) and $X^I_\pm$ are SU(N) singlets. The covariant derivative $\tilde{D}_\mu$ is defined by

$$\tilde{D}_\mu X^I = \hat{D}_\mu X^I - \frac{X_+\cdot X}{X_+^2}\partial_\mu X^I_+, \quad \hat{D}_\mu X^I = D_\mu X^I - X^I_+ B_\mu, \quad D_\mu X^I = \partial_\mu X^I + i[A_\mu, X^I] \tag{2.2}$$

and the SO(8) tensor $\tilde{Q}^{IJ}$ is given by

$$\tilde{Q}^{IJ} = S^{IJ} + \frac{X^I_+ X^J_+}{X_+^2}(\det S - 1), \qquad S^{IJ} = \delta^{IJ} + \frac{i}{\sqrt{T_2}}\frac{m^{IJ}}{\sqrt{X_+^2}}, \tag{2.3}$$

where m^{IJ} is expressed as

$$m^{IJ} = X_+^K M^{IJK}, \qquad M^{IJK} = X_+^I[X^J, X^K] + X_+^J[X^K, X^I] + X_+^K[X^I, X^J]. \quad (2.4)$$

In (2.1) $\tilde{Q}^{-1}_{IJ}$ denotes the matrix inverse of $\tilde{Q}^{IJ}$ and STr is the symmetrized trace [27]. The non-linear Lagrangian L is invariant under the obvious global SO(8) transformation and the SU(N) gauge transformation associated with the A_μ gauge field, and further the non-compact gauge transformation associated with the B_μ gauge field

$$\delta X^I = X_+^I \Lambda, \qquad \delta B_\mu = D_\mu \Lambda, \qquad \delta X_+^I = 0, \qquad \delta X_-^I = \mathrm{Tr}(X^I \Lambda). \quad (2.5)$$

The terms except for the first non-linear term and the second BF-coupling term in (2.1) are added to have consistency with the low-energy Lagrangian. In the non-linear Lagrangian L only the symmetric part of $\tilde{Q}^{-1}_{IJ}$ is taken into consideration.

We introduce a Lagrange multiplier p to rewrite the square root term in (2.1) as

$$\begin{aligned} &- T_2\sqrt{-\det\left(\eta_{\mu\nu} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ}\tilde{D}_\nu X^J\right)}(\det\tilde{Q})^{1/4} \\ &\qquad \to \left(\frac{T_2^2}{2p}\det(\eta_{\mu\nu} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ}\tilde{D}_\nu X^J) - \frac{p}{2}\right)(\det\tilde{Q})^{1/4}, \end{aligned} \quad (2.6)$$

where a matrix can be treated as a c-number within the symmetrized trace. Owing to $\tilde{Q}^{-1}_{IJ} = \tilde{Q}^{-1}_{JI}$ the relevant tensor is rearranged as

$$\eta_{\mu\nu} + \frac{1}{T_2}\tilde{D}_\mu X^I \tilde{Q}^{-1}_{IJ}\tilde{D}_\nu X^J = g_{\mu\nu} + \tilde{B}_\mu\tilde{B}_\nu, \quad (2.7)$$

where

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \frac{1}{T_2}\bar{D}_\mu X^I \tilde{P}_{IJ}\bar{D}_\nu X^J, \qquad \bar{D}_\mu X^I = D_\mu X^I - \frac{X_+\cdot X}{X_+^2}\partial_\mu X_+^I, \\ \tilde{B}_\mu &= \sqrt{\frac{X_+^I\tilde{Q}^{-1}_{IJ}X_+^J}{T_2}}\left(B_\mu - \frac{\bar{D}_\mu X^I\tilde{Q}^{-1}_{IJ}X_+^J}{X_+^K\tilde{Q}^{-1}_{KL}X_+^L}\right) \end{aligned} \quad (2.8)$$

with

$$\tilde{P}_{IJ} = \tilde{Q}^{-1}_{IJ} - \frac{\tilde{Q}^{-1}_{IK}X_+^K X_+^L\tilde{Q}^{-1}_{LJ}}{X_+^M\tilde{Q}^{-1}_{MN}X_+^N}, \quad (2.9)$$

which is orthogonal to X_+^I as $X_+^I\tilde{P}_{IJ} = 0$.

The expression (2.6) together with (2.7) is quadratic in B_μ so that the equation of motion for the auxiliary field B_μ is given by

$$g^{\mu\nu}\left(B_\nu - \frac{\bar{D}_\nu X^I\tilde{Q}^{-1}_{IJ}X_+^J}{X_+^K\tilde{Q}^{-1}_{KL}X_+^L}\right) = \frac{p}{T_2\det g(X_+^K\tilde{Q}^{-1}_{KL}X_+^L)(\det\tilde{Q})^{1/4}}\left(x^\mu - \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}\right), \quad (2.10)$$

where

$$x^\mu = \partial^\mu X_+^I P_{IJ} X^J \quad (2.11)$$

with a projection operator

$$P_{IJ} = \delta_{IJ} - \frac{X_+^I X_+^J}{X_+^2}. \quad (2.12)$$

Substituting the expression (2.10) back into the starting Lagrangian accompanied with the replacement (2.6) and solving the equation of motion for p we get

$$L = \mathrm{STr}\left[-T_2(\det\tilde{Q})^{1/4}\sqrt{-\det g}\sqrt{1+\frac{1}{2T_2(X_+^K\tilde{Q}_{KL}^{-1}X_+^L)\sqrt{\det\tilde{Q}}}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}}\right.$$
$$\left.+\frac{\bar{D}_\mu X^I\tilde{Q}_{IJ}^{-1}X_+^J}{X_+^K\tilde{Q}_{KL}^{-1}X_+^L}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}-x^\mu\right)\right]+L_0, \tag{2.13}$$

where

$$\mathcal{F}_{\mu\nu} = F_{\mu\nu}-\frac{1}{\det g}\epsilon_{\mu\nu\lambda}x^\lambda, \tag{2.14}$$

$$L_0 = \partial_\mu X_-^I\partial^\mu X_+^I-\mathrm{Tr}\left(\frac{X_+\cdot X}{X_+^2}D_\mu X^I\partial^\mu X_+^I-\frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2\partial_\mu X_+^I\partial^\mu X_+^I\right). \tag{2.15}$$

Here we use the identity for 3×3 matrices $g_{\mu\nu}+a\mathcal{F}_{\mu\nu}$ with $\mathcal{F}_{\mu\nu}=-\mathcal{F}_{\nu\mu}$

$$\det(g_{\mu\nu})\left(1+\frac{1}{2}a^2\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\right)=\det(g_{\mu\nu}+a\mathcal{F}_{\mu\nu}) \tag{2.16}$$

to obtain a DBI-type Lagrangian

$$L = \mathrm{STr}\left[-T_2(\det\tilde{Q})^{1/4}\sqrt{-\det\left(g_{\mu\nu}+\frac{1}{\sqrt{T_2(X_+^K\tilde{Q}_{KL}^{-1}X_+^L)(\det\tilde{Q})^{1/4}}}\mathcal{F}_{\mu\nu}\right)}\right.$$
$$\left.+\frac{\bar{D}_\mu X^I\tilde{Q}_{IJ}^{-1}X_+^J}{X_+^K\tilde{Q}_{KL}^{-1}X_+^L}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}-x^\mu\right)\right]+L_0. \tag{2.17}$$

The inverse matrix of $\tilde{Q}^{IJ}$ in (2.3) is given by

$$\tilde{Q}_{IJ}^{-1}=P_{IJ}+\frac{X_+^IX_+^J}{X_+^2}\frac{1}{\det S}+\left(\frac{m_0}{1-m_0}\right)^{IJ}, \tag{2.18}$$

where an orthogonal relation $m^{IJ}X_+^J=0$ is used and

$$\left(\frac{m_0}{1-m_0}\right)^{IJ} = m_0^{IJ}+(m_0^2)^{IJ}+(m_0^3)^{IJ}+\cdots,$$
$$m_0^{IJ} = -\frac{i}{\sqrt{T_2X_+^2}}m^{IJ} \tag{2.19}$$

with $(m_0^2)^{IJ}=m_0^{IK}m_0^{KJ}$. Since only the symmetric part of matrix $\tilde{Q}_{IJ}^{-1}$ is taken into account in the Lagrangian, the expression (2.18) is modified to be

$$\tilde{Q}_{IJ}^{-1}=P_{IJ}+\frac{X_+^IX_+^J}{X_+^2}\frac{1}{\det S}+\left(\frac{m_0^2}{1-m_0^2}\right)^{IJ}, \tag{2.20}$$

which obeys $\tilde{Q}^{-1}_{IJ} = \tilde{Q}^{-1}_{JI}$ and includes only terms with even number of M^{IJK} as expressed by

$$\left(\frac{m_0^2}{1-m_0^2}\right)^{IJ} = (m_0^2)^{IJ} + (m_0^4)^{IJ} + (m_0^6)^{IJ} + \cdots . \tag{2.21}$$

From this expression the following SO(8) invariant factors are simplified as

$$\begin{aligned} X_+^I \tilde{Q}^{-1}_{IJ} X_+^J &= X_+^2 \frac{1}{\det S}, \\ \bar{D}_\mu X^I \tilde{Q}^{-1}_{IJ} X_+^J &= \bar{D}_\mu X^I X_+^I \frac{1}{\det S} \end{aligned} \tag{2.22}$$

and the tensor $\tilde{P}_{IJ}$ in (2.9) is also given by

$$\tilde{P}_{IJ} = \tilde{Q}^{-1}_{IJ} - \frac{X_+^I X_+^J}{X_+^2} \frac{1}{\det S}. \tag{2.23}$$

The relations in (2.22) together with $\det \tilde{Q} = (\det S)^2$ make the DBI-type Lagrangian (2.17) a simple form

$$\begin{aligned} L = &-T_2 \mathrm{STr}\left(\sqrt{-\det\left(g_{\mu\nu} + \frac{1}{\sqrt{T_2 X_+^2}}\mathcal{F}_{\mu\nu}\right)}(\det S)^{1/2}\right) \\ &+\mathrm{Tr}\left(\frac{\bar{D}_\mu X^I X_+^I}{X_+^2}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - x^\mu\right)\right) + L_0, \end{aligned} \tag{2.24}$$

where $g_{\mu\nu}$ defined in (2.8) is rewritten by

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{T_2}\bar{D}_\mu X^I \left(P_{IJ} - \frac{1}{T_2 X_+^2}\left(\frac{m^2}{1+\frac{m^2}{T_2 X_+^2}}\right)^{IJ}\right)\bar{D}_\nu X^J \tag{2.25}$$

and there is a relation derived from (2.14)

$$\frac{1}{2}\epsilon^{\mu\nu\lambda}\mathcal{F}_{\nu\lambda} = \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - x^\mu. \tag{2.26}$$

Thus from the non-linear BF Lagrangian with two nonpropagating gauge fields A_μ, B_μ we have integrated the auxiliary B_μ gauge field to extract the DBI-type Lagrangian expressed in terms of the SU(N) dynamical A_μ gauge field.

Now to perform the low-energy expansion for the non-linear Lagrangian (2.24), we calculate $\det S^{IJ}$ for 8×8 matrices by making the $1/T_2$ expansion as

$$\det S = 1 + \frac{1}{2T_2 X_+^2}(m^2)^{II} - \frac{1}{4T_2^2 (X_+^2)^2}\left((m^4)^{II} - \frac{1}{2}((m^2)^{II})^2\right) + \cdots , \tag{2.27}$$

where $m^{IJ} = -m^{JI}$ is taken into account and $(m^2)^{II} = -X_+^2 M^{IJK} M^{IJK}/3$. There is the following identity with finite terms for any 3×3 matrices $A_{\mu\nu}$

$$\begin{aligned} \det(\eta_{\mu\nu} + A_{\mu\nu}) = \det\eta & \left(1 + \mathrm{tr}(\eta^{-1}A) - \frac{1}{2}\mathrm{tr}(\eta^{-1}A)^2 + \frac{1}{2}\left(\mathrm{tr}(\eta^{-1}A)\right)^2\right. \\ &\left. + \frac{1}{3}\mathrm{tr}(\eta^{-1}A)^3 - \frac{1}{2}\mathrm{tr}(\eta^{-1}A)\mathrm{tr}(\eta^{-1}A)^2\right), \end{aligned} \tag{2.28}$$

which gives the $1/T_2$ expansion for $\det g_{\mu\nu}$ in (2.13)

$$\det g_{\mu\nu} = -\left(1+\frac{1}{T_2}\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J + \frac{1}{T_2^2}\left(-\frac{1}{2}\bar{D}_\mu X^I P_{IJ}\bar{D}_\nu X^J \bar{D}^\nu X^K P_{KL}\bar{D}^\mu X^L \right.\right.$$
$$\left.\left. + \frac{1}{2}(\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J)^2 - \frac{1}{X_+^2}\bar{D}_\mu X^I m^{IK} m^{KJ}\bar{D}^\mu X^J\right) + O\left(\frac{1}{T_2^3}\right)\right). \quad (2.29)$$

We see that the SO(8) vectors $\bar{D}_\mu X^I$ are contracted with $(m^2)^{IJ}$ and the projection operator P^{IJ}. It is convenient to express the square root factor including $\mathcal{F}_{\mu\nu}$ in (2.13) in terms of $F^\mu \equiv \epsilon^{\mu\nu\lambda}F_{\nu\lambda}/2 - x^\mu$ which appears as an interaction $\bar{D}_\mu X^I X_+^I F^\mu / X_+^2$ in (2.24), and expand it through (2.25) and (2.29) as

$$\sqrt{1+\frac{1}{2T_2X_+^2}\mathcal{F}_{\mu\nu}\mathcal{F}_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}} = \sqrt{1+\frac{1}{T_2X_+^2\det g}F^\mu F^\nu g_{\mu\nu}} \quad (2.30)$$
$$= 1 - \frac{1}{2T_2X_+^2}F^\mu F^\nu \eta_{\mu\nu} + \frac{1}{2T_2^2X_+^2}\left(F_\mu F^\mu \bar{D}_\nu X^I P_{IJ}\bar{D}^\nu X^J \right.$$
$$\left. - F^\mu F^\nu \bar{D}_\mu X^I P_{IJ}\bar{D}_\nu X^J - \frac{1}{4X_+^2}(F_\mu F^\mu)^2\right) + O\left(\frac{1}{T_2^3}\right).$$

Gathering the expansions (2.27), (2.29) and (2.30) in (2.24) or (2.13) we obtain the low-energy effective Lagrangian whose leading part is given by

$$L = -NT_2 + \mathrm{Tr}\left(\frac{1}{12}M^{IJK}M^{IJK} - \frac{1}{2}\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J + \frac{1}{2X_+^2}F_\mu F^\mu + \frac{1}{X_+^2}\bar{D}_\mu X^I X_+^I F^\mu\right) + L_0, \quad (2.31)$$

where $F_\mu F^\mu/2X_+^2$ is alternatively expressed as $-f_{\mu\nu}f^{\mu\nu}/4X_+^2$ in terms of $f_{\mu\nu} \equiv F_{\mu\nu} + \epsilon_{\mu\nu\lambda}x^\lambda$. This leading Lagrangian shows the Janus field theory with a spacetime dependent coupling constant in ref. [11] (see [39]). This Lagrangian is rewritten by the following form

$$L = -NT_2 + \mathrm{Tr}\left(\frac{1}{12}M^{IJK}M^{IJK} - \frac{1}{2}D_\mu X^I P_{IJ}D^\mu X^J + \frac{1}{2X_+^2}X^I\partial^\mu X_+^I(X^J\partial_\mu X_+^J \right.$$
$$\left. -2D_\mu X^J X_+^J) - \frac{1}{4X_+^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2X_+^2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}(D_\mu X^I X_+^I - X^I\partial_\mu X_+^I)\right) + \partial_\mu X_-^I \partial^\mu X_+^I, \quad (2.32)$$

which is further compactly represented by

$$L = -NT_2 + \mathrm{Tr}\left(\frac{1}{12}M^{IJK}M^{IJK} - \frac{1}{2}D_\mu X^I D^\mu X^I \right.$$
$$\left. + \frac{1}{2X_+^2}\left(\frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} + D^\mu X^I X_+^I - X^I\partial^\mu X_+^I\right)^2\right) + \partial_\mu X_-^I\partial^\mu X_+^I. \quad (2.33)$$

The subleading terms of order $1/T_2$ are derived as

$$\frac{1}{8T_2}\mathrm{STr}\Bigg[\frac{1}{(X_+^2)^2}m^{IJ}m^{JK}m^{KL}m^{LI}-\frac{1}{36}\left((M^{IJK})^2\right)^2+2\bar{D}_\mu X^I P_{IJ}\bar{D}_\nu X^J\bar{D}^\nu X^K P_{KL}\bar{D}^\mu X^L$$
$$-\left(\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J\right)^2+\frac{1}{3}\left(M^{IJK}\right)^2\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J+\frac{4}{X_+^2}\bar{D}_\mu X^I m^{IK}m^{KJ}\bar{D}^\mu X^J$$
$$+\frac{(F_\mu F^\mu)^2}{(X_+^2)^2}-\frac{F_\mu F^\mu}{3X_+^2}\left(M^{IJK}\right)^2+\frac{4F_\mu F_\nu}{X_+^2}\bar{D}^\mu X^I P_{IJ}\bar{D}^\nu X^J-\frac{2F_\mu F^\mu}{X_+^2}\bar{D}_\nu X^I P_{IJ}\bar{D}^\nu X^J\Bigg]. \tag{2.34}$$

The last four terms including $F^\mu=\epsilon^{\mu\nu\lambda}f_{\nu\lambda}/2$ in (2.34) are expressed in terms of $f_{\mu\nu}$ as

$$\frac{1}{8T_2X_+^2}\mathrm{STr}\Bigg(\frac{1}{4X_+^2}(f_{\mu\nu}f^{\mu\nu})^2+4f_{\mu\nu}f_{\rho\sigma}\eta^{\mu\rho}\bar{D}^\nu X^I P_{IJ}\bar{D}^\sigma X^J$$
$$-f_{\mu\nu}f^{\mu\nu}\bar{D}_\lambda X^I P_{IJ}\bar{D}^\lambda X^J+\frac{1}{6}(M^{IJK})^2 f_{\mu\nu}f^{\mu\nu}\Bigg), \tag{2.35}$$

where a $f_{\mu\nu}$ is accompanied with a factor $1/\sqrt{X_+^2}$. The trace is taken symmetrically between all the matrix ingredients $f_{\mu\nu},\bar{D}_\mu X^I, M^{IJK}$ so that the expression (2.35) is described by

$$\frac{1}{12T_2X_+^2}\mathrm{Tr}\Bigg[-\frac{1}{2}(2f_{\mu\nu}f^{\mu\nu}\bar{D}_\lambda X^I\bar{D}^\lambda X^J+f_{\mu\nu}\bar{D}_\lambda X^I f^{\mu\nu}\bar{D}^\lambda X^J)P_{IJ} \tag{2.36}$$
$$+\left(2(f_\mu{}^\rho f^{\mu\nu}+f_\mu{}^\nu f^{\mu\rho})\bar{D}_\nu X^I\bar{D}_\rho X^J+f_\mu{}^\rho\bar{D}_\nu X^I f^{\mu\nu}\bar{D}_\rho X^J+f_\mu{}^\nu\bar{D}_\nu X^I f^{\mu\rho}\bar{D}_\rho X^J\right)P_{IJ}$$
$$+\frac{1}{8X_+^2}(2f_{\mu\nu}f^{\mu\nu}f_{\rho\sigma}f^{\rho\sigma}+f_{\mu\nu}f_{\rho\sigma}f^{\mu\nu}f^{\rho\sigma})+\frac{1}{12}(2f_{\mu\nu}f^{\mu\nu}(M^{IJK})^2+f_{\mu\nu}M^{IJK}f^{\mu\nu}M^{IJK})\Bigg].$$

The potential part in (2.34) is also expanded as

$$\frac{1}{24T_2}\mathrm{Tr}\Bigg[\frac{1}{(X_+^2)^2}\left((m^4)^{II}+2(m^2)^{IJ}(m^2)^{IJ}\right)$$
$$-\frac{1}{36}\left(M^{IJK}M^{LMN}M^{IJK}M^{LMN}+2((M^{IJK})^2)^2\right)\Bigg]. \tag{2.37}$$

Here we write down the remaining terms

$$\frac{1}{12T_2}\mathrm{Tr}\Bigg[\bar{D}_\mu X^I\bar{D}_\nu X^J\bar{D}^\nu X^K\bar{D}^\mu X^L+\bar{D}_\mu X^I\bar{D}_\nu X^K\bar{D}^\nu X^J\bar{D}^\mu X^L$$
$$+\bar{D}_\mu X^I\bar{D}_\nu X^K\bar{D}^\mu X^L\bar{D}^\nu X^J-\bar{D}_\mu X^I\bar{D}^\mu X^J\bar{D}_\nu X^K\bar{D}^\nu X^L$$
$$-\frac{1}{2}\bar{D}_\mu X^I\bar{D}_\nu X^K\bar{D}^\mu X^J\bar{D}^\nu X^L\Bigg]P_{IJ}P_{KL}$$

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$$+\frac{1}{12T_2}\mathrm{Tr}\left[\frac{2(m^2)^{IJ}}{X_+^2}(\bar{D}_\mu X^I\bar{D}^\mu X^J+\bar{D}_\mu X^J\bar{D}^\mu X^I)-\frac{1}{X_+^2}(\bar{D}_\mu X^I m^{IK}\bar{D}^\mu X^J m^{JK}\right.$$
$$+m^{KI}\bar{D}_\mu X^I m^{KJ}\bar{D}^\mu X^J)+\frac{1}{6}(2(M^{LMN})^2\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J$$
$$\left.+M^{LMN}\bar{D}_\mu X^I M^{LMN}\bar{D}^\mu X^J P_{IJ})\right]. \tag{2.38}$$

3 Two non-linear BF Lagrangians with even and odd number of M^{IJK}

There are propositions of two types of non-linear BF Lagrangians for multiple M2-branes, which include terms with even number as well as odd number of M^{IJK} [29]. One type is presented by

$$\begin{aligned}
L_1 &= -T_2\mathrm{STr}\left(\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T_2}\tilde{D}_\mu X^I\tilde{R}^{IJ}\tilde{D}_\nu X^J\right)}(\det S_1)^{1/4}\right) \\
&\quad+\frac{1}{2}\epsilon^{\mu\nu\lambda}\left(\mathrm{Tr}(B_\mu F_{\nu\lambda})-\frac{i}{T_2}\mathrm{STr}(\tilde{D}_\mu X^K\tilde{D}_\nu X^I M^{IKN}(S_1^{-1})^{NJ}\tilde{D}_\lambda X^J)\right) \\
&\quad+(\partial_\mu X_-^I-\mathrm{Tr}(X^I B_\mu))\partial^\mu X_+^I-\mathrm{Tr}\left(\frac{X_+\cdot X}{X_+^2}\hat{D}_\mu X^I\partial^\mu X_+^I-\frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2\partial_\mu X_+^I\partial^\mu X_+^I\right),
\end{aligned} \tag{3.1}$$

where the symmetric tensor $\tilde{R}^{IJ}$ is defined by

$$\begin{aligned}
\tilde{R}^{IJ} &= (S_1^{-1})^{IJ}+\frac{X_+^I X_+^J}{X_+^2}\left(\frac{1}{\sqrt{\det S_1}}-1\right), \\
S_1^{IJ} &= \delta^{IJ}-\frac{1}{T_2}M^{IKM}M^{JKN}\left(\frac{X_+^M X_+^N}{X_+^2}\right).
\end{aligned} \tag{3.2}$$

Because of $\det S_1=(\det S)^2$ the symmetric tensor $\tilde{R}^{IJ}$ is identical to $\tilde{Q}_{IJ}^{-1}$ in (2.20), and $(\det S_1)^{1/4}=(\det\tilde{Q})^{1/4}$, so that the Lagrangian L_1 except for terms with odd number of M^{IJK} reduces to L in (2.1). For this topological BF Lagrangian we consider the integration over the B_μ gauge field to obtain a dynamical gauge theory Lagrangian. Since the type one Lagrangian L_1 contains not only the mass term of B_μ but also the cubic term, we cannot perform the B_μ integration directly. Instead, we begin to make the low-energy expansion for the non-linear term in (3.1) up to $1/T_2$ order

$$\begin{aligned}
-T_2N+\mathrm{STr}&\left[-\frac{1}{2}\tilde{D}_\mu X^I\tilde{D}^\mu X^I+\frac{1}{4}A^{II}\right. \\
&\left.+\frac{1}{T_2}\left(-Z(B_\mu)+\frac{1}{8}(A^{II}\tilde{D}_\mu X^J\tilde{D}^\mu X^J+A^{IJ}A^{JI}-\frac{1}{4}(A^{II})^2)\right)\right],
\end{aligned} \tag{3.3}$$

where

$$A^{IJ}=M^{IKM}M^{JKN}\left(\frac{X_+^M X_+^N}{X_+^2}\right)=-\frac{1}{X_+^2}(m^2)^{IJ},$$

$$Z(B_\mu) = \frac{1}{8}\Bigg((\tilde{D}_\mu X^I \tilde{D}^\mu X^I)^2 - 2\tilde{D}_\mu X^I \tilde{D}_\nu X^I \tilde{D}^\nu X^J \tilde{D}^\mu X^J$$
$$+4\tilde{D}_\mu X^I \left(A^{IJ} + \frac{X_+^I X_+^J}{2X_+^2} A^{KK} \right) \tilde{D}^\mu X^J \Bigg). \tag{3.4}$$

The algebraic equation of motion for B_μ reads

$$X_+^I(\bar{D}^\mu X^I - X_+^I B^\mu) + \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - x^\mu$$
$$= \frac{1}{T_2}\left(\frac{1}{4}A^{II}X_+^J(\bar{D}^\mu X^J - X_+^J B^\mu) + \frac{\delta Z}{\delta B_\mu} + \frac{i}{2}\epsilon^{\rho\nu\lambda}\frac{\delta X_{\rho\nu\lambda}}{\delta B_\mu} \right) \tag{3.5}$$

with $X_{\rho\nu\lambda}(B^\mu) = \tilde{D}_\rho X^K \tilde{D}_\nu X^I M^{IKJ} \tilde{D}_\lambda X^J$. The solution can be iteratively derived by $B^\mu = B_0^\mu + B_1^\mu/T_2$, with

$$B_0^\mu = \frac{1}{X_+^2}\left(X_+^I \bar{D}^\mu X^I + \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - x^\mu \right) \tag{3.6}$$

and

$$B_1^\mu = -\frac{1}{X_+^2}\left(\frac{1}{4}A^{II}X_+^J(\bar{D}^\mu X^J - X_+^J B_0^\mu) + \frac{\delta Z}{\delta B_\mu}\Big|_{B_0^\mu} + \frac{i}{2}\epsilon^{\rho\nu\lambda}\frac{\delta X_{\rho\nu\lambda}}{\delta B_\mu}\Big|_{B_0^\mu} \right), \tag{3.7}$$

where the expression of B_0^μ (3.6) is inserted into the last two derivative terms. Substituting this solution back into the low-energy Lagrangian of L_1 (3.1) we obtain the same leading Lagrangian as (2.31) through a relation

$$\bar{D}^\mu X^I - X_+^I B_0^\mu = P^{IJ}\bar{D}^\mu X^J - \frac{1}{X_+^2}X_+^I F^\mu \tag{3.8}$$

and the following correction terms of order $1/T_2$

$$\frac{1}{T_2}\mathrm{STr}\Bigg((\bar{D}_\mu X^I - X_+^I B_{0\mu})X_+^I B_1^\mu + \frac{1}{8}A^{II}(\bar{D}_\mu X^J - X_+^J B_{0\mu})(\bar{D}^\mu X^J - X_+^J B_0^\mu)$$
$$+\frac{1}{8}(A^{IJ}A^{JI} - \frac{1}{4}(A^{II})^2) - Z(B_0^\mu) + F^\mu B_{1\mu} - \frac{i}{2}\epsilon^{\mu\nu\lambda}X_{\mu\nu\lambda}(B_0^\mu) \Bigg). \tag{3.9}$$

The subleading terms except for the terms including $X_{\mu\nu\lambda}(B_0^\mu)$ and $\delta X_{\rho\nu\lambda}/\delta B_\mu|_{B_0^\mu}$ are described by

$$\frac{1}{T_2}\mathrm{STr}\Bigg[\frac{1}{8}\left(A^{KK}\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J - A^{II}\frac{F_\mu F^\mu}{X_+^2} + A^{IJ}A^{JI} - \frac{1}{4}(A^{II})^2 \right)$$
$$+\frac{1}{4}\left(\bar{D}_\mu X^I P_{IJ}\bar{D}_\nu X^J \bar{D}^\nu X^K P_{KL}\bar{D}^\mu X^L + \frac{2F_\mu F_\nu}{X_+^2}\bar{D}^\mu X^I P_{IJ}\bar{D}^\nu X^J + \frac{(F_\mu F^\mu)^2}{(X_+^2)^2} \right)$$
$$-\frac{1}{8}\left((\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J)^2 + \frac{2F_\mu F^\mu}{X_+^2}\bar{D}_\nu X^I P_{IJ}\bar{D}^\nu X^J + \frac{(F_\mu F^\mu)^2}{(X_+^2)^2} \right)$$
$$+\frac{1}{2X_+^2}\bar{D}_\mu X^I m^{IK}m^{KJ}\bar{D}^\mu X^J \Bigg]. \tag{3.10}$$

It is noted that the SO(8) vectors $\bar{D}_\mu X^I$ are contracted with $(m^2)^{IJ}$ and the projection operator P_{IJ} which is due to (3.8). The expression (3.10) is confirmed to agree with (2.34). Thus we have observed that these subleading terms obtained by the iterative procedure for the B_μ integration in the low-energy Lagrangian reproduces the previous expression (2.34) which is derived by the low-energy expansion of the effective DBI-type Lagrangian generated by the exact B_μ integration.

The remaining terms lead to

$$\begin{aligned}
&-\frac{i}{2T_2}\mathrm{STr}\left(\epsilon^{\mu\nu\lambda}X_{\mu\nu\lambda}(B_0^\mu)+\epsilon^{\rho\nu\lambda}\frac{\delta X_{\rho\nu\lambda}}{\delta B_\mu}\Big|_{B_0^\mu}\left(\frac{X_+^I}{X_+^2}(\bar{D}_\mu X^I-X_+^I B_{0\mu})+\frac{1}{X_+^2}F_\mu\right)\right)\\
&\qquad=\frac{i}{2T_2}\epsilon^{\mu\nu\lambda}\mathrm{STr}\left(\tilde{M}^{IJK}\bar{D}_\mu X^I\bar{D}_\nu X^J\bar{D}_\lambda X^K-\frac{3m^{IJ}}{X_+^2}F_\mu\bar{D}_\nu X^I\bar{D}_\lambda X^J\right),
\end{aligned}\tag{3.11}$$

where $\tilde{M}^{IJK}$ is a totally antisymmetric tensor defined by

$$\tilde{M}^{IJK}=M^{IJK}-\frac{1}{X_+^2}\left(m^{IJ}X_+^K+m^{JK}X_+^I+m^{KI}X_+^J\right),\tag{3.12}$$

which is orthogonal to X_+^I as $\tilde{M}^{IJK}X_+^I=0$. This expression including single M^{IJK} is rewritten by

$$\frac{i}{2T_2}\mathrm{STr}(\epsilon^{\mu\nu\lambda}\tilde{M}^{IJK}\bar{D}_\mu X^I\bar{D}_\nu X^J\bar{D}_\lambda X^K+3m^{IJ}f^{\mu\nu}\bar{D}_\mu X^I\bar{D}_\nu X^J).\tag{3.13}$$

We see that there is a coupling between the gauge field strength $F^{\mu\nu}$ and $m^{IJ}\bar{D}_\mu X^I\bar{D}_\nu X^J$. The symmetrization in (3.13) is taken as

$$\begin{aligned}
&\frac{i}{2T_2}\mathrm{Tr}(\epsilon^{\mu\nu\lambda}\tilde{M}^{IJK}\bar{D}_\mu X^I\bar{D}_\nu X^J\bar{D}_\lambda X^K+m^{IJ}f^{\mu\nu}\bar{D}_\mu X^I\bar{D}_\nu X^J\\
&\qquad+m^{IJ}\bar{D}_\mu X^I f^{\mu\nu}\bar{D}_\nu X^J+m^{IJ}\bar{D}_\mu X^I\bar{D}_\nu X^J f^{\mu\nu}),
\end{aligned}\tag{3.14}$$

where the first term remains intact.

Now we turn to the other type of non-linear Lagrangian for non-Abelian BF membranes

$$\begin{aligned}
L_2\;=\;&-T_2\mathrm{STr}\left(\sqrt{-\det\left(\eta_{\mu\nu}+\frac{1}{T_2}\tilde{D}_\mu X^I(S_2^{-1})^{IJ}\tilde{D}_\nu X^J\right)}(\det S_2)^{1/6}\right)\\
&+\frac{1}{2}\epsilon^{\mu\nu\lambda}\left(\mathrm{Tr}(B_\mu F_{\nu\lambda})-\frac{i}{T_2}\mathrm{STr}(\tilde{D}_\mu X^K\tilde{D}_\nu X^I M^{IKN}(S_2^{-1})^{NJ}\tilde{D}_\lambda X^J)\right)\\
&+(\partial_\mu X_-^I-\mathrm{Tr}(X^I B_\mu))\partial^\mu X_+^I-\mathrm{Tr}\left(\frac{X_+\cdot X}{X_+^2}\hat{D}_\mu X^I\partial^\mu X_+^I-\frac{1}{2}\left(\frac{X_+\cdot X}{X_+^2}\right)^2\partial_\mu X_+^I\partial^\mu X_+^I\right),
\end{aligned}\tag{3.15}$$

where the symmetric tensor S_2^{IJ} is defined by

$$S_2^{IJ}=\delta^{IJ}-\frac{1}{2T_2}B^{IJ}\tag{3.16}$$

with $B^{IJ} = M^{IKM}M^{JKM} \equiv (M^2)^{IJ}$. We make the following low-energy expansion for the non-linear term in (3.15)

$$-T_2N + \mathrm{STr}\Bigg[-\frac{1}{2}\tilde{D}_\mu X^I \tilde{D}^\mu X^I + \frac{1}{12}B^{II} \\ +\frac{1}{T_2}\left(-W(B_\mu) + \frac{1}{24}\left(B^{II}\tilde{D}_\mu X^J \tilde{D}^\mu X^J + \frac{1}{2}B^{IJ}B^{JI} - \frac{1}{12}(B^{II})^2\right)\right)\Bigg], \quad (3.17)$$

where

$$W(B_\mu) = \frac{1}{8}\left((\tilde{D}_\mu X^I \tilde{D}^\mu X^I)^2 - 2\tilde{D}_\mu X^I \tilde{D}_\nu X^I \tilde{D}^\nu X^J \tilde{D}^\mu X^J + 2\tilde{D}_\mu X^I B^{IJ}\tilde{D}^\mu X^J\right). \quad (3.18)$$

The algebraic equation of motion for B_μ is also given by

$$X_+^I(\bar{D}^\mu X^I - X_+^I B^\mu) + \frac{1}{2}\epsilon^{\mu\nu\lambda}F_{\nu\lambda} - x^\mu \\ = \frac{1}{T_2}\left(\frac{1}{12}B^{II}X_+^J(\bar{D}^\mu X^J - X_+^J B^\mu) + \frac{\delta W}{\delta B_\mu} + \frac{i}{2}\epsilon^{\rho\nu\lambda}\frac{\delta X_{\rho\nu\lambda}}{\delta B_\mu}\right), \quad (3.19)$$

whose solution is iteratively derived by $B^\mu = B_0^\mu + \tilde{B}_1^\mu/T_2$ where B_0^μ is the same expression as (3.6) and $\tilde{B}_1^\mu$ is

$$\tilde{B}_1^\mu = -\frac{1}{X_+^2}\left(\frac{1}{12}B^{II}X_+^J(\bar{D}^\mu X^J - X_+^J B_0^\mu) + \frac{\delta W}{\delta B_\mu}\Big|_{B_0^\mu} + \frac{i}{2}\epsilon^{\rho\nu\lambda}\frac{\delta X_{\rho\nu\lambda}}{\delta B_\mu}\Big|_{B_0^\mu}\right). \quad (3.20)$$

The substitution of this solution into the low-energy Lagrangian of L_2 (3.15) yields the same leading Lagrangian as (2.31) and the following subleading terms of order $1/T_2$

$$\frac{1}{T_2}\mathrm{STr}\Bigg((\bar{D}_\mu X^I - X_+^I B_{0\mu})X_+^I \tilde{B}_1^\mu + \frac{1}{24}B^{II}(\bar{D}_\mu X^J - X_+^J B_{0\mu})(\bar{D}^\mu X^J - X_+^J B_0^\mu) \\ +\frac{1}{48}(B^{IJ}B^{JI} - \frac{1}{6}(B^{II})^2) - W(B_0^\mu) + F^\mu \tilde{B}_{1\mu} - \frac{i}{2}\epsilon^{\mu\nu\lambda}X_{\mu\nu\lambda}(B_0^\mu)\Bigg). \quad (3.21)$$

We use an identity $B^{II} = 3A^{II}$ to express (3.21) as sum of (3.11) and

$$\frac{1}{T_2}\mathrm{STr}\Bigg[\frac{1}{8}\left(A^{KK}\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J - A^{II}\frac{F_\mu F^\mu}{X_+^2} + \frac{1}{6}(M^2)^{IJ}(M^2)^{JI} - \frac{1}{4}(A^{II})^2\right) \\ +\frac{1}{4}\left(\bar{D}_\mu X^I P_{IJ}\bar{D}_\nu X^J \bar{D}^\nu X^K P_{KL}\bar{D}^\mu X^L + \frac{2F_\mu F_\nu}{X_+^2}\bar{D}^\mu X^I P_{IJ}\bar{D}^\nu X^J + \frac{(F_\mu F^\mu)^2}{(X_+^2)^2}\right) \\ -\frac{1}{8}\left((\bar{D}_\mu X^I P_{IJ}\bar{D}^\mu X^J)^2 + \frac{2F_\mu F^\mu}{X_+^2}\bar{D}_\nu X^I P_{IJ}\bar{D}^\nu X^J + \frac{(F_\mu F^\mu)^2}{(X_+^2)^2}\right) \\ -\frac{1}{4}\bar{D}_\mu X^I \bar{M}^{IMN}\bar{M}^{JMN}\bar{D}^\mu X^J\Bigg], \quad (3.22)$$

where $\bar{M}^{IMN} = P^{IK}M^{KMN}$ and a relation $\bar{M}^{IMN}m^{MN} = 0$ is used. The $1/T_2$ corrections show almost similar expressions to (3.10) with two different terms which are a potential term $(M^2)^{IJ}(M^2)^{JI}/6$ and an interaction term $-\bar{D}_\mu X^I \bar{M}^{IMN}\bar{M}^{JMN}\bar{D}^\mu X^J/4$.

4 Conclusion

Without resort to the low-energy expansion we have performed the integration over one Chern-Simons nonpropagating B_μ gauge field exactly for the non-linear Lagrangian of the BF membrane theory in ref. [26], which includes terms with even number of M^{IJK}. We have observed that there appears a non-linear DBI-type Lagrangian for the worldvolume theory of N M2-branes where the other Chern-Simons A_μ gauge field is promoted to the SU(N) dynamical propagating gauge field.

In the non-linear DBI-type Lagrangian the coefficient factor of the modified field strength $\mathcal{F}_{\mu\nu}$ takes a compact form $1/\sqrt{T_2 X_+^2}$ which yields the kinetic term of gauge field $-F_{\mu\nu}F^{\mu\nu}/4X_+^2$ with a space-time dependent coupling field X_+^I in the leading low-energy expansion. In the same way the linear term of F^μ also takes a compact interaction $\bar{D}_\mu X^I X_+^I F^\mu/X_+^2$. The subleading terms including dynamical gauge field strength $F_{\mu\nu}$ are expressed in terms of F^μ or alternatively $f_{\mu\nu}$ which is a specific combination of $F_{\mu\nu}$ and an SO(8) invariant contraction of scalar fields with the projection operator $\partial^\mu X_+^I P_{IJ} X^J$. In the subleading terms the SO(8) vectors $\bar{D}_\mu X^I$ are contracted with P^{IJ} and $(m^2)^{IJ}$ consisting of two M^{IJK}, which are orthogonal to X_+^I. This Lagrangian is regarded as the non-linear extension of the Janus field theory Lagrangian in ref. [11].

For the two types of non-linear BF Lagrangians in ref. [29] which include terms with even and odd number of M^{IJK}, we have made the low-energy expansion and then carried out the B_μ integration by solving its equation of motion in the presence of the $1/T_2$ order corrections through an iterative procedure. In the type one Lagrangian L_1 we have demonstrated that there appear indeed various terms at order $1/T_2$ in the iteratively B_μ integrated effective Lagrangian, but they except for terms with odd number of M^{IJK} are reshuffled to be in agreement with the $1/T_2$ order terms in the low-energy expansion of the above exactly B_μ integrated Lagrangian of the DBI form. In the type two Lagrangian L_2 the effective Lagrangian has been observed to have almost similar $1/T_2$ order corrections except for two terms, where $\bar{D}_\mu X^I$ are contracted with P^{IJ} as well as $\bar{M}^{IMN}\bar{M}^{JMN}$ which is also orthogonal to X_+^I. The remaining terms including single M^{IJK} in both effective Lagrangians consist of two kinds of interactions specified by $\epsilon^{\mu\nu\lambda}\tilde{M}^{IJK}\bar{D}_\mu X^I \bar{D}_\nu X^J \bar{D}_\lambda X^K$ and $m^{IJ} f^{\mu\nu}\bar{D}_\mu X^I \bar{D}_\nu X^J$ where the SO(8) vectors $\bar{D}_\mu X^I$ are contracted with the tensors $\tilde{M}^{IJK}$ and m^{IJ} which are orthogonal to X_+^I.

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